Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
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**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

# Thursday 08 October 2020

Afternoon | Paper Reference **8FM0/25**

# Further Mathematics
## Advanced Subsidiary
## Further Mathematics options
## 25: Further Mechanics 1
## (Part of options C, E, H and J)

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of $g$ is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

1. Two particles $P$ and $Q$ have masses $m$ and $4m$ respectively. The particles are at rest on a smooth horizontal plane. Particle $P$ is given a horizontal impulse, of magnitude $I$, in the direction $PQ$. Particle $P$ then collides directly with $Q$. Immediately after this collision, $P$ is at rest and $Q$ has speed $w$. The coefficient of restitution between the particles is $e$.

(a) Find $I$ in terms of $m$ and $w$.

**(2)**

(b) Show that $e = \dfrac{1}{4}$

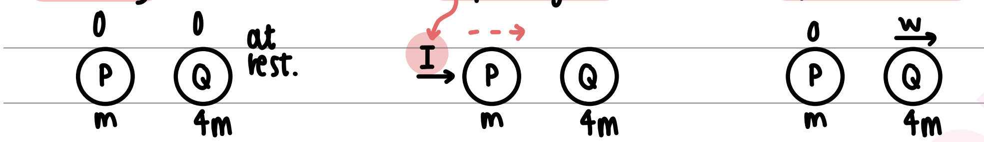
**(1)**

(c) Find, in terms of $m$ and $w$, the total kinetic energy lost in the collision between $P$ and $Q$.

**(2)**

Diagram

Initially: Impulse given: After Collision




(a) We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

initial velocity final velocity

→ Remember: Impulse = Δmomentum $\dfrac{\text{momentum}}{\text{mass}} = \text{speed}$

Substitute: $m \times \dfrac{I}{m} + 4m(0) = 4m(w) + m(0)$

$\cancel{m} \times \dfrac{I}{\cancel{m}} = 4mw$ M1

$I = 4mw$ A1

(b) Consider Q and use NLR

We can use Newton's Law of Restitution to get e.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution initial speed final speed

Substitute:

$e(u_p - 0) = w - 0$

$e = \dfrac{w}{u_p}$

To get $u_p$ consider impulse: $I = 4mw$, $4\cancel{m}w = \cancel{m}(u_p - 0)$ cancel m's

$4w = u_p$

$\therefore \ e = \dfrac{\cancel{w}}{4\cancel{w}}$ divide up + down by w.

hence $e = \frac{1}{4}$ shown B1

**Question 1 continued**

(c) Formula for KE lost:

$$\Delta KE = KE_{init} - KE_{fin.}$$

Substitute:

$$\Delta KE = \frac{1}{2}m(u_P)^2 + \cancelto{0}{\frac{1}{2}(4m)(u_Q)^2} - \cancelto{0}{\frac{1}{2}m(v_P)^2} - \frac{1}{2}(4m)(v_Q)^2 \quad \text{M1}$$

$$= \frac{1}{2}m(u_P)^2 - \frac{1}{2}(4m)(v_Q)^2$$

$$= \frac{1}{2}m(4w)^2 - \frac{1}{2}(4m)(w)^2$$

$$= 8mw^2 - 2mw^2$$

$$= 6mw^2 \text{ J lost} \quad \text{A1}$$

units for energy, Joules

**(Total for Question 1 is 5 marks)**

2. A car of mass 1000 kg moves along a straight horizontal road.

In all circumstances, when the speed of the car is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the car is modelled as a force of magnitude $cv^2$ N, where $c$ is a constant.

The maximum power that can be developed by the engine of the car is 50 kW.

At the instant when the speed of the car is $72\,\mathrm{km\,h^{-1}}$ and the engine is working at its maximum power, the acceleration of the car is $2.25\,\mathrm{m\,s^{-2}}$

(a) Convert $72\,\mathrm{km\,h^{-1}}$ into $\mathrm{m\,s^{-1}}$ **(1)**

(b) Find the acceleration of the car at the instant when the speed of the car is $144\,\mathrm{km\,h^{-1}}$ and the engine is working at its maximum power. **(7)**

The maximum speed of the car when the engine is working at its maximum power is $V\,\mathrm{km\,h^{-1}}$.

(c) Find, to the nearest whole number, the value of $V$. **(4)**

(a) 1km = 1000m

1h = 60min ; 1min = 60s ∴ 1h = 3600s (60min × 60s)

72km $\xrightarrow{\times 1000}$ 72 000 $\mathrm{mh^{-1}}$

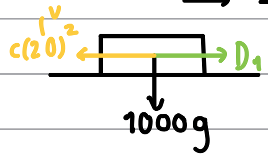
72 000 $\mathrm{mh^{-1}}$ $\xrightarrow{\div 3600}$ $20\,\mathrm{ms^{-1}}$

$72\,\mathrm{kmh^{-1}} = 20\,\mathrm{ms^{-1}}$ B1

**Question 2 continued**

(b) $144\,kmh^{-1} = 2 \times 72\,kmh^{-1} \therefore = 40\,ms^{-1}$ get speed in $ms^{-1}$

Diagram $v = 20\,ms^{-1}$ $2.25\,ms^{-2}$ $+\rightarrow$




To get $D_1$ we will use Power.

Formula for Power:

Power (W) $P = Dv$

Driving force (N) velocity ($ms^{-1}$)

$P = 50kW \xrightarrow{\times 1000} 50\,000W$ | $50\,00\cancel{0} = 2\cancel{0}D_1$

$D = D_1 N$ | $D_1 = \frac{5000}{2} = 2500N$ M1

$v = 20\,ms^{-1}$

Now we can use $\sum F_x = ma$ to get $c$

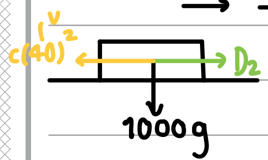
$D_1 - 400c = 1000 \times 2.25$ M1A1

$400c = 2500 - 2250$

$400c = 250$

$c = \frac{5}{8}$

Diagram $v = 40\,ms^{-1}$ $a\,ms^{-2}$ $+\rightarrow$




To get $D_2$ we will use Power.

Formula for Power:

Power (W) $P = Dv$

Driving force (N) velocity ($ms^{-1}$)

$P = 50kW \xrightarrow{\times 1000} 50\,000W$ | $50\,00\cancel{0} = 4\cancel{0}D_2$

$D = D_2 N$ | $D_2 = \frac{5000}{4} = 1250\,N$

$v = 40\,ms^{-1}$

Now we can use $\sum F_x = ma$ again now to get $a$ when $v = 40\,ms^{-1}$

$D_2 - \frac{5}{8}(40)^2 = 1000(a)$ M1A1

$1250 - \frac{5}{8}(1600) = 1000(a)$

$250 = 1000a$

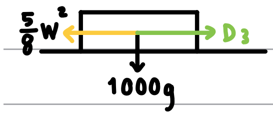
$a = 0.25\,ms^{-2}$ acceleration M1A1

**Question 2 continued**

(c) Diagram

$V\,kmh^{-1}$ → $W\,ms^{-1}$ convert units

$a=0$ as speed is max.




To get $D_3$ we will use Power.

Formula for Power:

Power (W) $P = Dv$ Driving force (N) velocity ($ms^{-1}$)

$$50000\,W = D_3 \times W \rightarrow D_3 = \frac{50000}{W}$$

Use $\Sigma F_x = 0$ as the speed is max. ∴ acceleration is 0.

$$\frac{50000}{W} - \frac{5}{8}W^2 = 0 \quad \text{M1A1}$$

$$50\,000 = \frac{5}{8}W^3$$

$$80\,000 = W^3$$

$$W = 43.088\,ms^{-1}$$

Now convert $Wms^{-1}$ to $Vkmh^{-1}$ by multiplying by 3.6. ( $\times 3600$ $s^{-1} \rightarrow h^{-1}$, $\div 1000$ m→km)

$$43.088 \times 3.6 \quad \text{M1}$$

$$= 155\,kmh^{-1} \text{ to 3sf} \quad \text{A1}$$

**Question 2 continued**

**(Total for Question 2 is 12 marks)**

**3.** Three particles $A$, $B$ and $C$ are at rest on a smooth horizontal plane. The particles lie along a straight line with $B$ between $A$ and $C$.

Particle $B$ has mass $4m$ and particle $C$ has mass $km$, where $k$ is a positive constant. Particle $B$ is projected with speed $u$ along the plane towards $C$ and they collide directly.

The coefficient of restitution between $B$ and $C$ is $\frac{1}{4}$

(a) Find the range of values of $k$ for which there would be no further collisions.

**(8)**

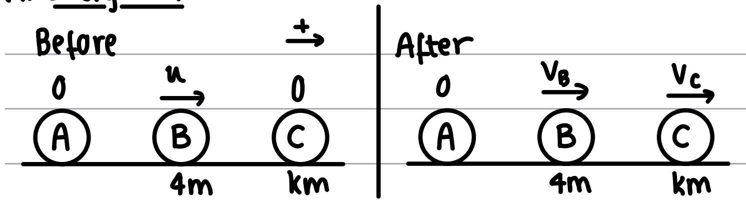
The magnitude of the impulse on $B$ in the collision between $B$ and $C$ is $3mu$

(b) Find the value of $k$.

**(4)**

**Question 3 continued**

(a) Diagram




draw out all 3 particles to visualize the situation. If one of the particles is not involved in a collision, don't include it in your calculation.

We can use the conservation of linear momentum to get an equation.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

initial velocity final velocity

Substitute: (only consider B and C)

$$4\cancel{m}u + k\cancel{m}(0) = 4\cancel{m}(v_B) + k\cancel{m}(v_C)$$ cancel m's M1A1

$$4u = 4v_B + kv_C \quad \text{Eq.1}$$

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution initial speed final speed

Substitute: (only consider B and C)

$$e(u - 0) = v_C - v_B \quad \text{M1A1}$$

$$\frac{u}{4} = v_C - v_B \quad \text{Eq.2}$$

Solve simultaneously Eq1 and Eq2:

so that we can eliminate $v_C$ use elimination method

$$4u = 4v_B + kv_C$$

$$\frac{u}{4} = v_C - v_B \;(\times -k) \qquad -\frac{ku}{4} = kv_B - kv_C \quad +$$

$$4u - \frac{ku}{4} = (4+k)v_B$$

$$16u - ku = 4(4+k)v_B$$

M1A1 $$\frac{u(16-k)}{4(4+k)} = v_B$$ speed of B after

For no further collisions to not occur, $v_B$ needs to not change direction, $\therefore v_B \geq 0$.

$$\frac{u(16-k)}{4(k+4)} \geq 0 \quad \text{M1}$$

$$u(16-k) \geq 0$$

$$k \leq 16$$

We are told that k is positive.

$\therefore$ $0 < k \leq 16$ range for k A1



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**Question 3 continued**

(b) Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta \text{momentum} = m v_{final} - m v_{initial}$$

mass

velocity

Substitute:

the impulse on B acts in the negative direction ∴ consider sign

$$-3mu = 4m(v_B - u) \quad \text{M1A1}$$

we will consider particle B since we already have its speed from (a)

$$-3\cancel{m}u = 4\cancel{m}\left(\frac{u(16-k)}{4(4+k)} - u\right)$$

$$-3\cancel{u} = 4\cancel{u}\left(\frac{16-k}{4(4+k)} - 1\right)$$

cancel u's

$$-\frac{3}{4} = \frac{16-k}{4(4+k)} - 1$$

solve for k

$$\frac{1}{4} = \frac{16-k}{4(4+k)}$$

$$4 + k = 16 - k$$

$$2k = 12$$

$$k = 6$$

value of k

M1A1

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**Question 3 continued**

**(Total for Question 3 is 12 marks)**

**4.** A small ball, of mass $m$, is thrown vertically upwards with speed $\sqrt{8gH}$ from a point $O$ on a smooth horizontal floor. The ball moves towards a smooth horizontal ceiling that is a vertical distance $H$ above $O$. The coefficient of restitution between the ball and the ceiling is $\frac{1}{2}$

In a model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to air resistance of constant magnitude $\frac{1}{2}mg$.

Using this model,

(a) use the work-energy principle to find, in terms of $g$ and $H$, the speed of the ball immediately before it strikes the ceiling,

**(5)**

(b) find, in terms of $g$ and $H$, the speed of the ball immediately before it strikes the floor at $O$ for the first time.

**(5)**

In a simplified model of the motion of the ball, it is assumed that the ball, as it moves up or down, is subject to no air resistance.

Using this simplified model,

(c) explain, without any detailed calculation, why the speed of the ball, immediately before it strikes the floor at $O$ for the first time, would still be less than $\sqrt{8gH}$

**(1)**

★ Work-Energy Principle: an increase of KE/GPE is caused by an equal amount of positive work done on the body(e.g. engine) and a decrease of KE/GPE is caused by an equal amount of negative work done on the body(e.g. friction).

★ Remember the work-energy formulae:

Either: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} = \underbrace{KE_F}_{\text{final kinetic}} + \overbrace{GPE_F}^{\text{final grav. potential}} + \underbrace{\text{WD against friction}}_{\text{work lost to friction}}$

OR: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} - \underbrace{\text{WD by friction}}_{\text{we subtract this since it leaves the system as heat!}} = \underbrace{KE_F}_{\text{final kinetic}} + \overbrace{GPE_F}^{\text{final grav. potential}}$

★ Formulae for KE and GPE:

$KE = \frac{1}{2}\underbrace{m}_{\text{mass}}\overbrace{v^2}^{\text{velocity}}$

$GPE = \underbrace{m}_{\text{mass}}\underbrace{g}_{g=9.8ms^{-2}}\overbrace{h}^{\text{change in height}}$



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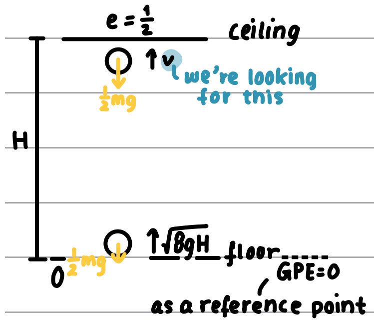
**Question 4 continued**

(a) Diagram




Substitute:

$$\underbrace{\tfrac{1}{2}m(\sqrt{8gH})^2 + mg(0)}_{\text{at floor}} - \underbrace{\tfrac{1}{2}mg \times H}_{\text{work done against air resistance}} = \underbrace{\tfrac{1}{2}mv^2 + mgH}_{\text{at ceiling}}$$ M1

solve for this!

B1B1 $\frac{1}{2}\cancel{m}(8gH) - \frac{1}{2}\cancel{m}gH - \cancel{m}gH = \frac{1}{2}\cancel{m}v^2$ cancel m's A1

$4gH - \frac{1}{2}gH - gH = \frac{1}{2}v^2$

$4gH - \frac{3}{2}gH = \frac{1}{2}v^2$

$8gH - 3gH = v^2$

$5gH = v^2$

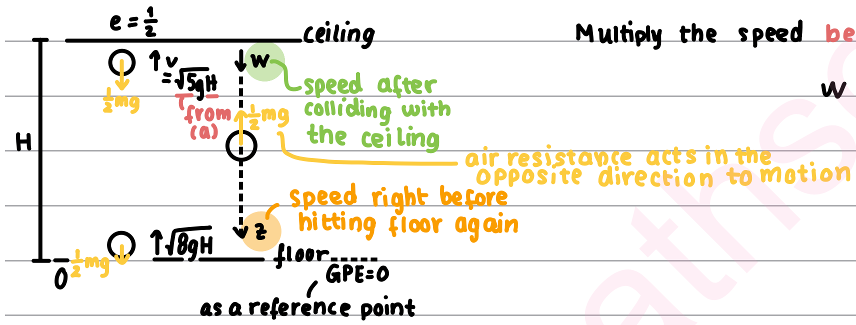
$v = \sqrt{5gH}$ speed at the ceiling

A1

★ remember: we don't consider the weight as an extra force, since GPE already considers it!

($\therefore$ no "work done against gravity"!)

(b) Diagram




$\rightarrow$ let's start by getting w. We will do this using NLR.

Multiply the speed before (v) by e to get w.

$w = \frac{1}{2}v = \frac{1}{2}\sqrt{5gH}$ M1

this is the speed after the collision with the wall. We will use this in our calculations.

Method 1 – use work–energy principle

★ check above for an explanation and for formulae.

Substitute:

$$\tfrac{1}{2}m\left(\tfrac{1}{2}\sqrt{5gH}\right)^2 + mgH - \tfrac{1}{2}mgH = \tfrac{1}{2}mz^2 + \cancel{mg(0)}^{\,0}$$ M1A1

$\frac{5}{8}\cancel{m}gH + \frac{1}{2}\cancel{m}gH = \frac{1}{2}\cancel{m}z^2$ cancel m

$\frac{9}{8}gH = \frac{1}{2}z^2$ A1

$\frac{9}{4}gH = z^2 \longrightarrow z = \frac{3}{2}\sqrt{gH}\ \text{ms}^{-1}$ speed before it hits floor A1

Method 2 – use suvat with $a = \frac{g}{2}$ ($\downarrow +$) M1

$s = H$

$u = \frac{1}{2}\sqrt{5gH} = w$

$v = z$

$a = \frac{g}{2}$

$t$ ✗

Use formula

$v^2 = u^2 + 2as$

$z^2 = \left(\frac{1}{2}\sqrt{5gH}\right)^2 + \cancel{2}\left(\frac{g}{\cancel{2}}\right)(H)$ A1A1

$z^2 = \frac{1}{4}(5gH) + gH$

$z^2 = \frac{9}{4}gH \longrightarrow z = \frac{3}{2}\sqrt{gH}\ \text{ms}^{-1}$ speed before it hits floor A1

(c) Since $e < 1$, the ball lost energy due to its collision with the ceiling B1

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 11 marks)**

**TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS**